

# Baryogenesis in false vacuum

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## Abstract

The null result in the LHC may indicate that the standard model is not drastically modified up to very high scale such as the GUT/string scale. Having this in the mind, we suggest a novel leptogenesis scenario realized in the false vacuum of the Higgs field. If the Higgs field develops the large vacuum expectation value in the early universe, the lepton number violating process is enhanced, which we use for baryogenesis. To demonstrate the scenario, several models are discussed. For example, we show that the observed baryon asymmetry is successfully generated in the standard model with a second Higgs doublet and a singlet scalar.

## I. INTRODUCTION

Although the standard model (SM) is complete after the discovery of the Higgs boson at the Large Hadron Collider [1, 2], there are still mysteries in elementary particle physics such as the finite neutrino mass and dark matter. Besides them the baryon asymmetry in the universe (BAU) is also one of unsolved problems. That is, how had baryogenesis been realized in evolution of the universe? The latest cosmological result from the Planck observations [3] tells us that the BAU is

$$\frac{n_B}{s} = (8.67 \pm 0.05) \times 10^{-11}, \quad (1)$$

where  $n_B$  is the baryon number density and  $s$  is the entropy density.

In order to theoretically explain the BAU within elementary particle physics, the Sakharov conditions [4] have to be satisfied: There exists a process violating the baryon number;  $C$  and  $CP$  invariances are violated; the system leaves from equilibrium state. The SM does not accommodate the departure from equilibrium. Although the baryon number is violated through the sphaleron process and  $CP$  symmetry is violated in the weak interaction, it is not enough to reproduce the BAU. Therefore the SM cannot satisfy these conditions and must be extended.

Some baryogenesis mechanisms satisfying the Sakharov conditions have been suggested, e.g. the grand unified theory [5] and the Affleck–Dine mechanism [6]. Leptogenesis is also one of the well-known mechanisms for baryogenesis [7] (see also reviews [8, 9]) where we use the fact that through the sphaleron process [10–13], the difference  $B - L$  between the baryon number  $B$  and the lepton number  $L$  is conserved whereas their sum  $B + L$  is not. The baryon number density in thermal equilibrium is provided by the  $B - L$  number density via the sphaleron process:

$$n_B = \frac{8N_F + 4N_S}{22N_F + 13N_S} n_{B-L}, \quad (2)$$

where  $N_F$  is generation of quarks and leptons and  $N_S$  is that of scalar doublets. For instance, in case of the SM where  $N_F = 3$  and  $N_S = 1$ , the factor in the right-hand side is  $28/79$ . Through the decay of the heavy particle, the lepton number is generated, and then its number density changes to the  $B - L$  number density  $n_{B-L}$ , whose process is described by the coupled Boltzmann equations for these number densities.

In this paper, we study the leptogenesis realized in the false vacuum of the Higgs field. The mass of the particles coupled to the Higgs field becomes massive. There are the lepton number violating processes where the heavy particles decay into the lighter ones. At the same time, the phase transition of the Higgs takes place and the Higgs moves from the false vacuum to the true electroweak one.

To demonstrate this scenario, two models are investigated. We first consider a minimal model depending on the SM with a high dimensional operator,

$$\Delta\mathcal{L}_5 = \frac{\lambda_{ij}}{\Lambda} HH \bar{L}_j^c L_i, \quad (3)$$

where  $L_i$  is the lepton doublet,  $\Lambda$  is a cutoff scale with  $\mathcal{O}(10^{14})$  GeV,<sup>1</sup> and the Higgs doublet is defined as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ h + i\chi_3 \end{pmatrix}. \quad (4)$$

Such a operator is generated in typically the type I seesaw model by integrating out the right-hand neutrino. This effective interaction breaks the lepton number and thus is used as the source of the lepton asymmetry. In particular, we consider the decay of the left-hand neutrino, which is given by the mode  $\nu \rightarrow \ell^- W^+$ . Note that in the broken phase  $\langle H \rangle \neq 0$ , this operator turns to a neutrino mass term,

$$\frac{\langle H \rangle^2}{\Lambda} (\bar{\nu}^c \nu + \bar{\nu} \nu^c) = \frac{\langle h \rangle^2}{2\Lambda} (\bar{\nu}^c \nu + \bar{\nu} \nu^c), \quad (5)$$

where we have assumed that the coupling constant  $\lambda_{ij}$  is of order one since neutrino can have a finite mass  $m_\nu \sim 0.1$  eV. Therefore, the leptogenesis takes place in the false vacuum where the neutrino mass  $\langle h \rangle^2 / \Lambda$  becomes larger value than the charged lepton and  $W$  boson ones. As will be seen in next section, in such a minimal model, the baryon asymmetry produced by this process actually is not adequate for the observed value (1).

Next, we consider an extended system in which the second Higgs doublet is introduced. In this case, we will see that the lepton asymmetry is caused by the second Higgs boson and it is possible to explain the observation.

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<sup>1</sup> In Ref. [14, 15], the operator (3) is used to realize leptogenesis as well as the  $CP$  violating operator  $\bar{L}_i \gamma^\mu L_j \bar{L}_i \gamma_\mu L_j$ . These operators are naturally generated in a low energy effective theory of various seesaw models.

We have to see whether or not the phase transition of the Higgs field from the false vacuum to the electroweak one occurs after the lepton asymmetry is produced. To this end, we investigate the thermal history of the Higgs potential. Including a new singlet-scalar field coupled to the SM Higgs field, there exists a certain parameter space where the phase transition appropriately takes place.

We organize this paper as follow: In next section, we present the formulation of the Boltzmann equations in order to calculate the baryon asymmetry. Numerically solving them, we investigate the produced baryon asymmetry for two cases explained above. Section III is devoted to investigate the thermal history of the Higgs potential. We summarize and discuss our study and obtained results, and we comment on the possibility of the high scale electroweak baryogenesis in section IV. In appendix A, the thermal effects on the Higgs potential and their formulations are shown.

## II. MECHANISM AND BOLTZMANN EQUATIONS

First, we consider the situation where the decay of the left-handed neutrino produces the baryon asymmetry. In this section, we present the Boltzmann equations and quantitatively evaluate the baryon asymmetry by numerically solving them. We evaluate the baryon asymmetry produced by the left-handed neutrino decay; however we see that not enough baryon asymmetry is produced. To ameliorate the situation, next we add the second Higgs doublet. We demonstrate that the decay of new charged Higgs boson can reproduce the observed amount of asymmetry.

### A. The derivation of Boltzmann equations

In this subsection, to calculate the asymmetry of the universe, we follow Ref. [9, 16, 17] and derive the Boltzmann equations for general case of leptogenesis. The change of the

number density of a heavy particle is governed by

$$\begin{aligned}
\dot{n}_X + 3Hn_X = & \int d\Pi_X d\Pi_1 d\Pi_2 (2\pi)^4 \delta^{(4)}(p_X - p_1 - p_2) \\
& \times \left( -f(p_X) |\mathcal{M}(X \rightarrow 12)|^2 + f(p_1) f(p_2) |\mathcal{M}(12 \rightarrow X)|^2 \right) \\
& + \int d\Pi_X d\Pi_Y d\Pi_1 d\Pi_2 \cdots d\Pi_N (2\pi)^4 \delta^{(4)}(p_X + p_Y - p_1 - p_2 - \cdots - p_N) \\
& \times \left( -f(p_X) f(p_Y) |\mathcal{M}(XY \rightarrow 12 \cdots N)|^2 + f(p_1) \cdots f(p_N) |\mathcal{M}(12 \cdots N \rightarrow XY)|^2 \right),
\end{aligned} \tag{6}$$

where  $X$  and  $Y$  represent the heavy particles without lepton number; the number  $1 \cdots N$  denotes lighter particle; the dot on  $n_X$  in the left-hand side denotes the time derivative; we have neglected the effects of the Pauli blocking and stimulated emission;  $d\Pi_i = d^3p_i/(2\pi)^3 2E_i$  is the phase space integral;  $H = \dot{R}/R$  is the Hubble parameter given by the scale factor  $R$  which is governed by the Friedmann equation.  $f$  is the distribution function, and especially in the case where the system is in thermal equilibrium,  $f$  is given as the Maxwell-Boltzmann distribution.

The first and second terms of the right-hand side in Eq. (6) correspond to the decay and annihilation of heavy particle, respectively. Let us rewrite the first term by using the definition of the decay rate,

$$\Gamma_X \equiv \frac{1}{2E_X} \int d\Pi_1 d\Pi_2 (2\pi)^4 \delta^{(4)}(p_X - p_1 - p_2) |\mathcal{M}(X \rightarrow 12)|^2. \tag{7}$$

We use the fact that the kinetic equilibrium allows us to make the replacement,<sup>2</sup>

$$f(p_1) f(p_2) = f^{\text{EQ}}(p_1) f^{\text{EQ}}(p_2) = f^{\text{EQ}}(p_X). \tag{8}$$

Furthermore, at the leading order,  $|\mathcal{M}(X \rightarrow 12)|^2 = |\mathcal{M}(12 \rightarrow X)|^2$ . Hence, we find that the first term in the right-hand side becomes

$$\left( -n_X + n_X^{\text{EQ}} \right) \Gamma_X. \tag{9}$$

The second term in Eq. (6) can be written in terms of the thermal average cross section of the pair annihilation  $\langle \sigma_{\text{ann}} v \rangle$ :

$$\langle \sigma_{\text{ann}} v \rangle = \frac{\int d\Pi_X d\Pi_Y d\Pi_1 \cdots d\Pi_N (2\pi)^4 \delta^{(4)}(p_X + p_Y - p_1 - \cdots - p_N) |\mathcal{M}(XY \rightarrow 1 \cdots N)|^2}{\int d\Pi_X d\Pi_Y (2E_X)(2E_Y) f(p_X) f(p_Y)}. \tag{10}$$

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<sup>2</sup> Here we neglect the chemical potential of particle 1 and 2 as the effect is subleading.

We assume that  $f(p_i) \propto f^{\text{EQ}}(p_i)$  thanks to the kinetic equilibrium, so that the second term in Eq. (6) becomes

$$\langle \sigma_{\text{ann}} v \rangle \left( -n_X^2 + \left( n_X^{\text{EQ}} \right)^2 \right). \quad (11)$$

To summarize, the Boltzmann equation of  $n_X$  is given by

$$\dot{n}_X + 3Hn_X = \left( -n_X + n_X^{\text{EQ}} \right) \Gamma_X + \langle \sigma_{\text{ann}} v \rangle \left( -n_X^2 + \left( n_X^{\text{EQ}} \right)^2 \right). \quad (12)$$

In a similar manner, we can write the Boltzmann equation governing the lepton number density:

$$\begin{aligned} \dot{n}_l + 3Hn_l = & \int d\Pi_X d\Pi_l d\Pi_Y \delta^{(4)}(p_X - p_l - p_Y) \\ & \times \epsilon \left( -f(p_X) |\mathcal{M}(X \rightarrow lW)|^2 + f(p_l) f(p_Y) |\mathcal{M}(lW \rightarrow X)|^2 \right) \\ & + 2 \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\ & \times \left( -f(p_1) f(p_2) |\mathcal{M}(l_1 l_2 \rightarrow l_3 l_4)|^2 + f(p_3) f(p_4) |\mathcal{M}(l_3 l_4 \rightarrow l_1 l_2)|^2 \right) \end{aligned} \quad (13)$$

where the first and second terms in the right-hand side describe the decay of the heavy particle and annihilation of the leptons, respectively;  $W$  is a particle without the lepton number;  $l_i$  is a particle having the lepton number. Furthermore, we rewrite this equation as one for  $B - L$  asymmetry, which is given by

$$\dot{n}_{B-L} + 3Hn_{B-L} = -\epsilon \left( n_X - n_X^{\text{EQ}} \right) \Gamma_X \text{Br} - n_{B-L} \Gamma_X \text{Br} \frac{n_X^{\text{EQ}}}{n_\gamma} - 2n_{B-L} n_l \langle \sigma_L v \rangle, \quad (14)$$

where  $\epsilon$  is the parameter which denotes the  $CP$  asymmetry;  $\text{Br}$  is the branching ratio of  $X \rightarrow lW$ ;  $n_\gamma$  is the number density of photon; and  $\langle \sigma_L v \rangle$  is the thermally-averaged scattering cross section which does not conserve the lepton number.

It is convenient to introduce  $N_i \equiv n_i/n_\gamma$  because this quantity is conserved under the cosmic expansion. We also introduce  $z \equiv M_X/T$  as a variable. Using these variables, let us now rewrite the Boltzmann equations. For instance, the left-hand side becomes

$$\dot{n}_X + 3Hn_X = n_\gamma \dot{N}_X = n_\gamma H z \frac{d}{dz} N_X, \quad (15)$$

where in the second equality, we have used

$$\frac{dT}{dt} = -3H \frac{n_\gamma}{dn_\gamma/dT} = -HT. \quad (16)$$

The right-hand side is

$$\begin{aligned}
& -\Gamma_X(z) \left( n_X - n_X^{\text{EQ}} \right) - \langle \sigma_{\text{ann}} v \rangle \left( n_X^2 - \left( n_X^{\text{EQ}} \right)^2 \right) \\
& = -n_\gamma H z \left( \frac{\Gamma_X(z)}{H(z)z} \right) (N_X - N_X^{\text{EQ}}) - n_\gamma H z \left( \frac{\langle \sigma_{\text{ann}} v \rangle n_\gamma}{H(z)z} \right) \left( N_X^2 - \left( N_X^{\text{EQ}} \right)^2 \right). \quad (17)
\end{aligned}$$

In terms of  $N_i$  and  $z$ , we can write the set of the Boltzmann equations as follows:

$$\frac{d}{dz} N_X = - \left( \frac{\Gamma_X(z)}{H(z)z} \right) (N_X - N_X^{\text{EQ}}) - \left( \frac{\langle \sigma_{\text{ann}} v \rangle n_\gamma}{H(z)z} \right) \left( N_X^2 - \left( N_X^{\text{EQ}} \right)^2 \right), \quad (18)$$

$$\begin{aligned}
\frac{d}{dz} N_{B-L} = & - \left( \frac{\epsilon \Gamma_X(z) \text{Br}}{H(z)z} \right) (N_X - N_X^{\text{EQ}}) - N_{B-L} \left( \frac{\Gamma_X(z) \text{Br}}{H(z)z} \right) N_X^{\text{EQ}} \\
& - \left( \frac{\langle \sigma_L v \rangle n_\gamma}{H(z)z} \right) 2N_{B-L} N_l, \quad (19)
\end{aligned}$$

$$H^2(z) = \frac{\pi^2 g_*(z)}{90} \frac{z^4 M_X^4}{M_P^2}, \quad (20)$$

$$N_X^{\text{EQ}} = \frac{g}{4\zeta(3)} z^2 K_2(z), \quad (21)$$

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3 = \frac{2\zeta(3)}{\pi^2} z^{-3} M_X, \quad (22)$$

where  $\zeta(3) \approx 1.20205$  is the Riemann zeta function of 3;  $M_P = \sqrt{\hbar c / 8\pi G} = 2.435 \times 10^{18}$  GeV is the reduced Planck scale;  $K_2$  is the modified Bessel functions of second kind;  $g_*(z)$  is the total number of effectively massless degree of freedom; and  $g$  is the internal degree of freedom of the heavy particle. We neglect the  $z$  dependence of  $g_*(z)$  and use  $g_* = 106.75$ .

Simultaneously solving the Boltzmann equations, we can evaluate the value of the lepton asymmetry due to the decay of left-handed neutrino which is identified as the heavy particle  $X$ . In order to perform numerical calculations, we have to specify  $\Gamma_X, \text{Br}, \epsilon, \langle \sigma_{\text{ann}} v \rangle$  and  $\langle \sigma_L v \rangle$ . In next subsection, we give these variables for the minimal model.

## B. Minimal model case

We evaluate the baryon asymmetry in the minimal model whose Lagrangian is given as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \Delta\mathcal{L}_5, \quad (23)$$

where  $\mathcal{L}_{\text{SM}}$  is the Lagrangian of the SM and  $\Delta\mathcal{L}_5$  is the higher dimensional operator given in Eq. (3). The lepton number is produced by the decay of the left-handed neutrino. We now show the variables given in the Boltzmann equations in order.

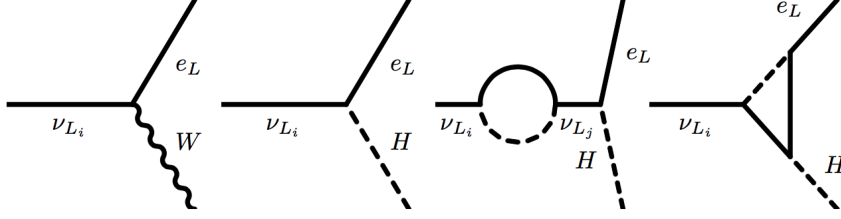


FIG. 1: The diagrams which contribute to the asymmetry by the decay of the left-hand neutrino. The last diagram gives the complex phase only if the mass of neutrino is larger than that of  $W$  boson.

The masses of left-handed neutrino, Higgs boson and  $W$  boson in the broken phase  $\langle H \rangle \neq 0$  are given as

$$M_\nu = \frac{\langle h \rangle^2}{\Lambda}, \quad M_H = \sqrt{2\lambda} \langle h \rangle, \quad M_W = \frac{1}{2} g_2 \langle h \rangle, \quad (24)$$

respectively, where  $\lambda$  is the quartic coupling constant of the Higgs field, and  $g_2$  is the  $SU(2)_L$  gauge coupling constant

The decay rate of the left-handed neutrino and the branching ratio to the longitudinal gauge boson are calculated as

$$\Gamma_X(z) = \left\langle \frac{1}{\gamma} \right\rangle \Gamma_X|_{z=\infty} = \left\langle \frac{1}{\gamma} \right\rangle \frac{M_\nu}{8\pi} \left( \left( \frac{\langle h \rangle}{\sqrt{2}\Lambda} \right)^2 + g_2^2 \right), \quad \text{Br} \simeq \frac{y_\tau^2}{\left( \frac{\langle h \rangle}{\sqrt{2}\Lambda} \right)^2 + g_2^2}, \quad (25)$$

where  $\langle 1/\gamma \rangle = K_1(z)/K_2(z)$  in the thermal bath,  $K_1$  is the modified Bessel functions of first kind, and  $y_\tau$  is the tau Yukawa coupling. We note that the branching ratio to the transverse gauge boson is important. This is because, in order to pick up the imaginary part of the amplitude, it is needed to use lepton Yukawa coupling rather than  $SU(2)$  gauge coupling.

The  $CP$  asymmetry  $\epsilon$  comes from the interference between the tree and loop diagram shown in Fig. 1, whose order is given by

$$\epsilon_i \simeq \frac{1}{8\pi} \sum_j \frac{\text{Im} \left[ (Y Y^\dagger)_{ij}^2 \right]}{y_\tau^2}, \quad (26)$$

$m_{\nu,i}$  represents the left handed neutrino mass in our electroweak vacuum, and  $Y$  is the charged lepton Yukawa matrix. Note that the imaginary part appears only if  $M_\nu > M_W +$



$M_\tau$ ,<sup>3</sup> which yields

$$\langle h \rangle > \frac{g_2}{2} \Lambda \simeq 1.5 \times 10^{14} \text{ GeV} \left( \frac{\Lambda}{6 \times 10^{14} \text{ GeV}} \right). \quad (27)$$

Let us estimate the imaginary parts of the Yukawa coupling constants in Eq. (26). The numerator of Eq. (26) is related to the Jarlskog invariant in the lepton sector [19], and the order is estimated as [19]

$$\epsilon_i \sim \frac{1}{8\pi} y_\tau^2 (3 \times 10^{-2} \sin \delta) \simeq 1.2 \times 10^{-7} \left( \frac{y_\tau}{10^{-2}} \right)^2 \left( \frac{\sin \delta}{1} \right), \quad (28)$$

where  $\delta$  is Dirac CP phase of neutrino sector.

We note that, by using the renormalization group equations, we obtain the values of coupling constants at the high scale:<sup>4</sup>

$$g_2 \simeq 0.25, \quad y_\tau \simeq 1 \times 10^{-2}, \quad \alpha_2 \simeq \frac{g_2^2}{4\pi} = \frac{1}{50}, \quad (29)$$

#### *Numerical result in minimal model*

The Planck observation [3] tells us

$$N_{B,\text{obs}} \simeq 6.1 \times 10^{-10} \times \frac{2387}{86} = 1.7 \times 10^{-8}, \quad (30)$$

where the factor 2387/86 is the photon production factor. If this value comes from the sphaleron effect, we should have

$$N_{B-L,\text{obs}} \simeq 6.1 \times 10^{-10} \times \frac{2387}{86} \times \frac{79}{28} = 4.8 \times 10^{-8}. \quad (31)$$

Therefore, we numerically solve the Boltzmann equations given in Eq. (18)–(22) and investigate whether or not the appropriate parameter space which satisfies the value (31) exists.

Unfortunately, we can easily see that the baryon asymmetry cannot be reproduced in this framework. We obtain

$$\epsilon_i \text{Br} \sim 1.2 \times 10^{-7} \frac{y_\tau^2}{\left( \frac{\langle h \rangle}{\sqrt{2}\Lambda} \right)^2 + g_2^2} \left( \frac{y_\tau}{10^{-2}} \right)^2 \left( \frac{\sin \delta}{1} \right) \lesssim 2 \times 10^{-10}, \quad (32)$$

by combining Eq. (25) and Eq. (28), and hence the resultant baryon asymmetry is too small to explain current data. This indicates the necessity of extension of the model. In the next subsection, we present the possible extension, namely the extension with new Higgs doublet.

<sup>3</sup> Even if  $M_\nu < M_W + M_\tau$ , the imaginary part appears in higher order. However, it is too small to obtain the enough baryon asymmetry.

<sup>4</sup> See e.g. Ref. [18].

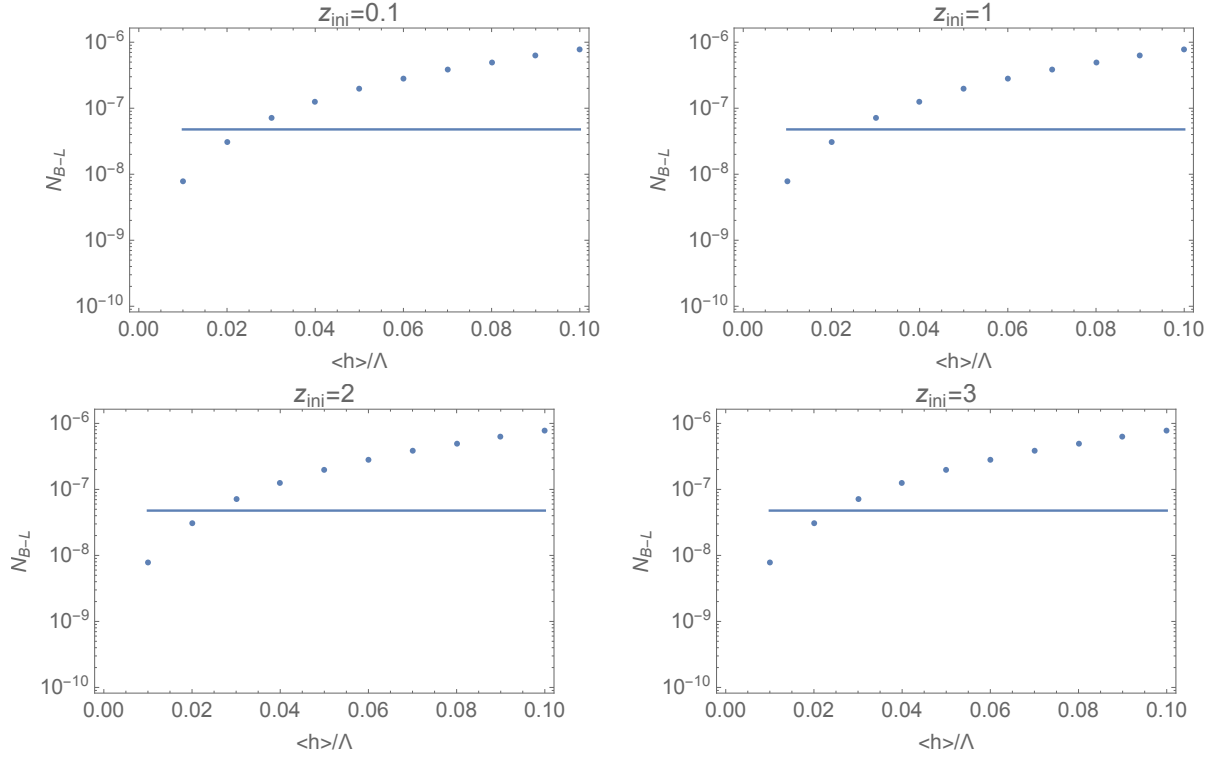


FIG. 2:  $N_{B-L}$  created by the decay of charged Higgs. Here we take the mass of the heavy Higgs as  $0.7 \langle h \rangle$  and the heaviest neutrino mass to be  $0.05 \text{ eV}$ , and plot the value of  $N_{B-L}$  at  $z = 5$ .

### C. Extended model: Two Higgs doublets

A way to improve the situation is to add new particle. If the second Higgs doublet  $H_2$  is introduced, we have new interaction terms:

$$\Delta\mathcal{L} = y_{2,ij} \bar{E}_i H_2 L_j + \dots \quad (33)$$

Then, the decay of the charged Higgs boson can generate the baryon asymmetry. The diagram contributing to the asymmetry is shown in Fig. 3. In this case, the decay rate is

$$\Gamma_X(z) = \left\langle \frac{1}{\gamma} \right\rangle \frac{M_{H_2}}{8\pi} \left( y_2^2 + \frac{\langle h \rangle^2}{2\Lambda^2} \right), \quad (34)$$

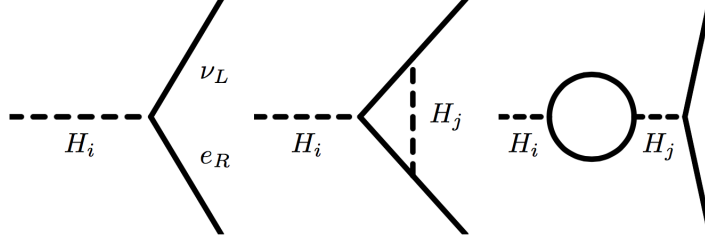


FIG. 3: The diagram which contributes the asymmetry by the decay of the charged Higgs boson.

and the variables are given by

$$\begin{aligned}
\epsilon &\simeq \sum_{ij} \frac{\text{Im} \left[ \left( y_2 y_2^\dagger \right)^2 \right]_{ij}}{8\pi \left( y_2 y_2^\dagger \right)_{ij}} \frac{M_\nu}{M_{H_2}}, & \langle \sigma_{\text{ann}} v \rangle &\simeq \alpha_2^2 \frac{1}{\text{Max}(M_{H_2}^2, T^2)}, \\
\langle \sigma_L v \rangle &\simeq \left( \frac{y_2^2}{4\pi} \right)^2 \frac{1}{\text{Max}(M_{H_2}^2, T^2)}, & \text{Br} &\simeq \frac{1}{1 + \langle h \rangle^2 / (2y_2^2 \Lambda^2)},
\end{aligned} \tag{35}$$

where  $M_{H_2}$  is the mass of the second Higgs doublet.

We evaluate the resultant asymmetry obtained by the decay of the SM Higgs. The set of the initial conditions of the Boltzmann equations is

$$N_X(z_{\text{ini}}) = 0, \quad N_{B-L}(z_{\text{ini}}) = 0, \tag{36}$$

and we set the cutoff scale as  $\Lambda = 6 \times 10^{14}$  GeV. In Fig. 2, we show the result assuming that  $y_{2,ij} \sim 1$  and the  $CP$  phase is of the order of one, i.e.  $e^{i\delta} \sim 1$ . We can see that the BAU is reproduced in this extension. In this model, the mass of  $H_2$  should be around  $10^{13-14}$  GeV in order to obtain consistent thermal history, as we will see in later. Note that since the new Higgs boson is added, the factor in Eq. (2) slightly changes. The factor becomes 8/23 for  $N_F = 3$  and  $N_S = 2$ .

### III. THERMAL HISTORY

In this section, we discuss the thermal history of the universe. We introduce a new scalar  $S$  to make Higgs field stay at false vacuum in the early universe, where  $S$  is singlet under the SM gauge group.

First, we explain the zero temperature scalar potential of the extended model with  $S$  and the thermal correction to it. Then, we discuss how the Higgs field is in false vacuum in the early universe.

### A. Zero temperature Higgs potential

The tree level scalar potential is given by

$$V_{\text{tree}}(h, S) = -\kappa \frac{m_S^2}{4\lambda_S} h^2 + \frac{1}{4} \lambda h^4 + \kappa h^2 S^2 - \frac{1}{2} m_S^2 S^2 + \lambda_S S^4, \quad (37)$$

where  $S$  is the new singlet scalar field. We omit the field  $H_2$  since it is not relevant to discussion here. We consider the region where all couplings take  $\mathcal{O}(0.1\text{--}1)$  value. Although  $\lambda$  becomes small or negative at high scale in the SM (see e.g. Ref. [20]), now the running of  $\lambda$  is modified,  $\lambda$  can take  $\mathcal{O}(0.1\text{--}1)$  value since some scalar fields are added.

We note that the one-loop Coleman–Weinberg potential can be safely neglected because of  $\mathcal{O}(0.1\text{--}1)$  couplings, and therefore we do not include it for simplicity.

The potential (37) has an absolute minimum at<sup>5</sup>

$$\langle h \rangle = 0, \quad \langle S \rangle = \frac{1}{2} \sqrt{\frac{m_S^2}{\lambda_S}} \equiv v_S. \quad (38)$$

The quadratic term of the SM Higgs is added in order to make the Higgs massless in this vacuum.

### B. Thermal potential

We follow the Ref. [21] and show the thermal potentials. The thermal potentials are evaluated at the one-loop level where the loop effects of the massive Higgs boson,  $W$ ,  $Z$  boson, the top quark and the scalar  $S$  are included. For the gauge fields, we employ the Landau gauge where the ghost fields are massless and do not have the  $h$  field dependence. The NG bosons  $\chi_i$  in the Higgs doublet field (4) are neglected since their effects are small.

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<sup>5</sup> The potential (37) has a minimum at  $\langle h \rangle = \sqrt{\frac{\kappa m_S^2}{2\lambda\lambda_S}} = \sqrt{\frac{2\kappa}{\lambda}} v_S$ ,  $\langle S \rangle = 0$ . This minimum does not become the absolute minimum but the local one for the parameter space we consider here.

As the thermal effects, there are two components, namely  $V_{\text{FT}}(h, T)$  and  $V_{\text{ring}}(h, T)$ .<sup>6</sup> The main contribution of thermal effects comes from  $V_{\text{FT}}(h, T)$ , which is

$$V_{\text{FT}}(h, T) = \frac{T^4}{2\pi^2} \left[ J_B(\tilde{m}_S^2/T^2) + J_B(\tilde{m}_h^2/T^2) + 6J_B(m_W^2/T^2) + 3J_B(m_Z^2/T^2) - 12J_F(m_t^2/T^2) \right], \quad (39)$$

where the mass for each particle is given by

$$\begin{aligned} m_W^2 &= \frac{g_2^2}{4} h^2, & m_Z^2 &= \frac{g_2^2 + g_Y^2}{4} h^2, & m_t^2 &= \frac{y_t^2}{2} h^2, \\ \tilde{m}_h^2 &= 3\lambda h^2 - \kappa \frac{m_S^2}{2\lambda_S} + 2\kappa S^2, & \tilde{m}_S^2 &= 12\lambda_S S^2 + 2\kappa h^2 - m_S^2; \end{aligned} \quad (40)$$

the thermal functions are defined as

$$J_B(r^2) = \int_0^\infty dx x^2 \ln \left( 1 - e^{-\sqrt{x^2 + r^2}} \right), \quad J_F(r^2) = \int_0^\infty dx x^2 \ln \left( 1 + e^{-\sqrt{x^2 + r^2}} \right). \quad (41)$$

Remember here that the coupling constants  $g_2$ ,  $g_Y$  and  $y_t$  are  $\text{SU}(2)_L$ ,  $\text{U}(1)_Y$  and top-Yukawa coupling constants, respectively. Since one cannot analytically and exactly evaluate these functions, the approximated expressions are made.<sup>7</sup>

There are contributions to the ring diagrams (or the daisy diagrams) from the Higgs boson and the gauge boson:

$$\begin{aligned} V_{\text{ring}}(h, T) &= -\frac{T}{12\pi} \left[ (\tilde{m}_h^2 + \Pi_h(T))^{3/2} - \tilde{m}_h^3 \right] - \frac{T}{12\pi} \left[ (\tilde{m}_S^2 + \Pi_h(T))^{3/2} - \tilde{m}_S^3 \right] \\ &\quad - \frac{T}{12\pi} \left[ 2a_g^{3/2} + \frac{1}{2\sqrt{2}} (a_g + c_g - [(a_g - c_g)^2 + 4b_g^2]^{1/2})^{3/2} \right. \\ &\quad \left. + \frac{1}{2\sqrt{2}} (a_g + c_g + [(a_g - c_g)^2 + 4b_g^2]^{1/2})^{3/2} - \frac{1}{4} [g_2^2 h^2]^{3/2} - \frac{1}{8} [(g_2^2 + g_Y^2) h^2]^{3/2} \right], \end{aligned} \quad (42)$$

where the first and second terms correspond to the contribution from the Higgs and the

<sup>6</sup> The derivation of these functions is shown in appendix A.

<sup>7</sup> The high temperature expansion is often used. However, they are not useful for the case where we see the large field value of  $h$ . Therefore, the fitting functions (A23) are also employed [22]. See appendix A for detail.

scalar  $S$ ;<sup>8</sup> the thermal masses of the Higgs and scalar  $S$  are

$$\Pi_h(T) = \frac{T^2}{12} \left( \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 + 3y_t^2 + 6\lambda + \frac{\kappa}{12} \right), \quad (44)$$

$$\Pi_S(T) = \frac{T^2}{12} \left( \frac{\lambda_S}{4} + \frac{\kappa}{3} \right); \quad (45)$$

and we have defined

$$a_g = \frac{1}{4}g_2^2h^2 + \frac{11}{6}g_2^2T^2, \quad b_g = -\frac{1}{4}g_2g_Yh^2, \quad c_g = \frac{1}{4}g_Y^2h^2 + \frac{11}{6}g_Y^2T^2. \quad (46)$$

To summarize, in order to trace the thermal history of the Higgs potential in the SM, we analyze the effective potential

$$V_{\text{eff}}(h, S, T) = V_{\text{tree}}(h, S) + V_{\text{FT}}(h, S, T) + V_{\text{ring}}(h, S, T), \quad (47)$$

where  $V_{\text{tree}}(h, S)$  is given in Eq. (37). In next subsection, we investigate the phase transition of Higgs field by using this potential.

### C. Thermal history

In the early universe, due to the finite temperature effect,  $S$  and  $H$  does not have the vacuum expectation value(VEV).<sup>9</sup> They develop the VEV at the temperature when the thermal mass term becomes comparable with the negative mass term. By utilizing the high temperature expansion (A21) and (A22), we estimate the critical temperatures which are given as the vanishing curvature of  $V_{\text{eff}}(h, S, T)$  at the origin  $(h, S) = (0, 0)$ , namely

$$\left. \frac{\partial^2 V_{\text{eff}}(h, S, T_S)}{\partial S^2} \right|_{h=0, S=0} = 0, \quad \left. \frac{\partial^2 V_{\text{eff}}(h, S, T_h)}{\partial h^2} \right|_{h=0, S=0} = 0. \quad (48)$$

Solving these equations for  $T$ , we find

$$T_S = \frac{2\sqrt{3}\sqrt{2v_s^2\lambda_S}}{\sqrt{\kappa + 6\lambda_S}}, \quad T_h = \frac{4\sqrt{6}\sqrt{v_s^2\kappa}}{\sqrt{9g_2^2 + 3g_Y^2 + 12y_t^2 + 8\kappa + 12\lambda}}. \quad (49)$$

---

<sup>8</sup> Combining the ring contribution of the Higgs boson and the first term of Eq. (39), we can write

$$\frac{T^4}{2\pi^2}J_B(\tilde{m}_h^2/T^2) - \frac{T}{12\pi} \left[ (\tilde{m}_h^2 + \Pi_h(T))^{3/2} - \tilde{m}_h^3 \right] = \frac{T^4}{2\pi^2}J_B(\tilde{m}_h^2(T)/T^2), \quad (43)$$

where  $\tilde{m}^2(T) = \tilde{m}_h^2 + \Pi_h(T)$  is the Debye mass of the Higgs boson. In the same manner, the thermal effects for the scalar  $S$  also can be written as the same form.

<sup>9</sup> Our thermal scenario is similar with Ref. [23] where the gravitational wave from electroweak phase transition at the high scale is discussed.

Here  $T_S$  and  $T_h$  denote the critical temperatures of the phase transition of  $S$  and  $h$ , respectively. Our scenario is as follows. The phase transition of Higgs field happens at  $T = T_h$ . At this time,  $S$  and  $h$  are in the false vacuum,  $\langle S \rangle = 0$ ,  $\langle h \rangle = \sqrt{\frac{2\kappa}{\lambda}}v_S$ , and the lepton number is created by the decay of heavy charged Higgs. After that, at  $T = T_S$ ,  $S$  develops VEV  $\langle S \rangle = v_S$ , and then  $\langle h \rangle$  comes back to the true vacuum Eq. (38).

In order to work our scenario, we require

$$T_S < T_h. \quad (50)$$

Moreover,  $S$  must have the negative mass at  $\langle S \rangle = 0$ ,  $\langle h \rangle = \sqrt{\frac{2\kappa}{\lambda}}v_S$ , namely  $\tilde{m}_S < 0$  which yields

$$\lambda_S > \frac{\kappa^2}{\lambda}. \quad (51)$$

As an example of successful parameters, we take  $\kappa = 0.7$ ,  $\lambda_S \simeq 1.5$ ,  $\lambda = 0.4$  and  $\langle h \rangle = 2 \times 10^{13} \text{ GeV}$ .  $T_h$  and  $T_S$  become

$$T_S \simeq 1.9v_S, \quad T_h \simeq 2.0v_S, \quad (52)$$

and the Eq. (51) is satisfied. Here  $g_Y = g_2 = y_t = 0.5$  is used.

Therefore, by solving the Boltzmann equations with

$$z_{\text{ini}} = \frac{M_{H_2}}{T_h}, \quad z_{\text{final}} = \frac{M_{H_2}}{T_S}, \quad (53)$$

we can calculate the asymmetry. For example, we obtain<sup>10</sup>

$$N_{B-L} \simeq 2.6 \times 10^{-7}, \quad (54)$$

with  $|y_{2,ij}| = 1$ ,  $M_{H_2} = 1.5v_S$  and the  $CP$  phase being one. We note that  $M_{H_2}$  should be close to the temperature of phase transition, otherwise the decay of  $H_2$  is not effective. Here we have taken into account the washout factor [15] in the symmetric phase,

$$\exp[-T_S/2 \times 10^{13} \text{ GeV}]. \quad (55)$$

This implies that we can realize the observed value,  $N_{B-L, \text{obs}} = 4.8 \times 10^{-8}$ , by slightly changing the value of  $CP$  phase.

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<sup>10</sup> Here we take the thermal initial condition,  $N_X = 3/4$ ,  $N_{B-L} = 0$ .

#### IV. SUMMARY AND DISCUSSION

We have considered the possibility of the baryogenesis in the false vacuum where the Higgs field develops the large field value compared with the electroweak scale. Since all the SM particles receive mass from the coupling with the Higgs boson, the large field value of the Higgs field means that they are super heavy. We have estimated the asymmetry produced by the decay of the heavy left-handed neutrino. It has turned out that the decay of neutrino can not realize the observed baryon asymmetry. If the new Higgs doublets  $H_2$  is introduced, the decay of new charged Higgs boson can provide the enough asymmetry.

We have also presented the thermal history where the Higgs develops the large field value in the early universe. It has been found that, by adding the singlet scalar  $S$ , our scenario safely works.

Finally, we briefly mention the possibility of the high scale electroweak baryogenesis. So far, we pursue the possibility that the baryon asymmetry is created by the heavy particle while the lepton number violation is given by Majorana mass term of left-handed neutrino. However, if the coupling  $\lambda$  is small, the electroweak phase transition at high scale becomes first-order. Since our extended model has many  $CP$  phases, there is chance to generate the  $B + L$  asymmetry. If the  $L$  asymmetry is washed out in the false vacuum, the net  $B$  asymmetry survives. The condition of  $L$  wash out would be roughly given by

$$\left(\frac{\langle H \rangle}{\Lambda}\right)^2 \frac{1}{8\pi} M_W \gtrsim \sqrt{10} \frac{T^2}{M_P}. \quad (56)$$

By putting  $T \simeq \langle H \rangle$ ,  $M_W \simeq \langle H \rangle$ , we obtain

$$\langle H \rangle \gtrsim 1 \times 10^{13} \text{ GeV}. \quad (57)$$

Hence, we have chance to create the baryon asymmetry by the electroweak baryogenesis in addition to the decay of heavy particle.

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## Appendix A: The effective potential at finite temperature

In this appendix, following Ref. [21], we show the derivation of the thermal effects on the Higgs potential in the SM. We consider the one-loop contribution of a particle with the mass  $m(h)$  to the potential, which typically has the following form:

$$V_{\text{1loop}}(h, T) = \pm \frac{1}{2} \text{Tr} \ln(k^2 + m^2(h)), \quad (\text{A1})$$

where “Tr” denotes the functional trace;  $k$  is the Euclidean momentum; the boson (fermion) loop case has overall positive (negative) sign. For a particle with one degree of freedom, the potential is

$$V_{\text{1loop}}(h, T) = \pm \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \ln(k^2 + m^2(h)). \quad (\text{A2})$$

At finite temperature, the time direction of momentum is discretized and its loop integral changes to the Matsubara summation:

$$\int \frac{dk_0}{2\pi} \int \frac{d^3 k}{(2\pi)^3} f(k_0, \vec{k}) = T \sum_{n=-\infty}^{\infty} \int \frac{d^3 k}{(2\pi)^3} f(\omega_n, \vec{k}), \quad (\text{A3})$$

with the Matsubara frequency,

$$\omega_n = \begin{cases} 2n\pi T & \text{for boson,} \\ (2n+1)\pi T & \text{for fermion.} \end{cases} \quad (\text{A4})$$

Therefore, the Eq. (A2) can be calculated as

$$\begin{aligned} V_{\text{1loop}}(h, T) &= \pm \frac{T}{2} \int \frac{d^3 k}{(2\pi)^3} [\beta\omega + 2 \ln(1 \mp e^{-\beta\omega})] \\ &= \pm \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \omega \pm T \int \frac{d^3 k}{(2\pi)^3} \ln(1 \mp e^{-\beta\omega}), \end{aligned} \quad (\text{A5})$$

where  $\omega = \sqrt{\vec{k}^2 + m^2}$ ; the sign (+) and (−) in the logarithm apply to fermions and bosons, respectively. The first term does not depend on temperature and is rewritten as

$$\begin{aligned} V_{\text{CW}}(h) &\equiv \pm \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \omega \\ &= \pm \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \ln(k^2 + m^2). \end{aligned} \quad (\text{A6})$$

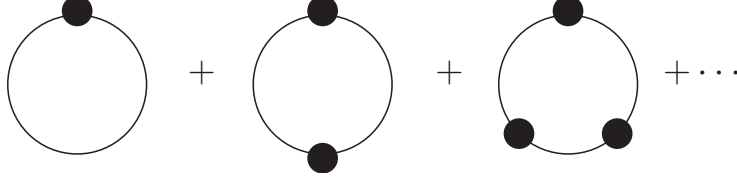


FIG. 4: The ring diagrams. The black circle denotes the propagator with loop corrections.

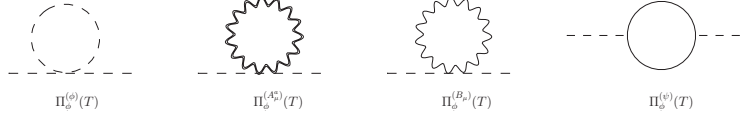


FIG. 5: The two-point functions of Higgs field at one-loop level which yield the thermal masses. The dashed, single wave, double wave and solid lines denote the Higgs,  $U(1)_Y$  gauge,  $SU(2)_L$  and top-quark, respectively.

This is one-loop contribution at vanishing temperature, i.e. the Coleman–Weinberg potential. The second term is the thermal potential at one-loop level and becomes

$$V_{\text{FT}}(h, T) \equiv \pm \frac{T}{2\pi^2} \int dk k^2 \ln(1 \pm e^{-\beta\omega}) = \pm \frac{T^4}{2\pi^2} J_{B(F)}(r^2), \quad (\text{A7})$$

where the thermal functions for boson and fermion are defined as

$$J_B(r^2) = \int_0^\infty dx x^2 \ln(1 - e^{-\sqrt{x^2+r^2}}), \quad J_F(r^2) = \int_0^\infty dx x^2 \ln(1 + e^{-\sqrt{x^2+r^2}}), \quad (\text{A8})$$

with  $x \equiv |\vec{k}|/T$  and  $r \equiv m(h)/T$ . Note that in general case, the operator  $k^2 + m^2(h)$  is not diagonal, i.e.  $k^2\delta_{ij} + m_{ij}^2(h)$ . Therefore, the mass matrix  $m_{ij}^2(h)$  has to be diagonalized.

In the SM case, taking account of the degrees of freedom of particles, the thermal potential is given by

$$V_{\text{FT}}(h, T) = \sum_{i=W, Z, h} n_i \frac{T^4}{2\pi^2} J_B((m_i^B(h)/T)^2) - \sum_{i=t} n_i \frac{T^4}{2\pi^2} J_F((m_i^F(h)/T)^2), \quad (\text{A9})$$

where  $n_W = 6$ ,  $n_Z = 3$ ,  $n_t = 12$  and  $n_h = 1$ .

Next, we consider the ring (or daisy) contributions shown in Fig. 4, which are the next-higher-order corrections and are related to the infrared divergence; see e.g. [24] for a detailed

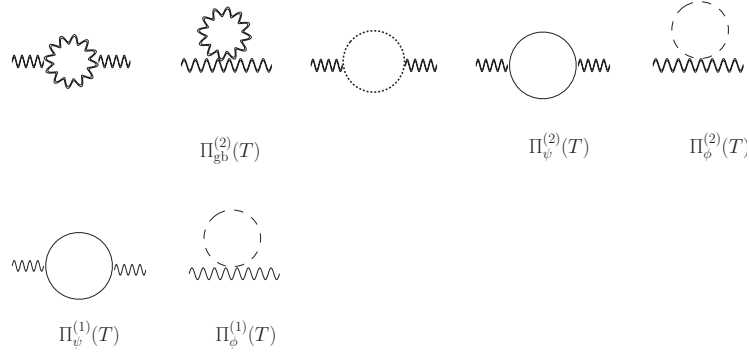


FIG. 6: The two-point functions of SU(2) and U(1) gauge fields at one-loop level which yield the thermal masses. The dashed, single wave, double wave, solid and dot lines denote the Higgs, U(1)<sub>Y</sub> gauge, SU(2)<sub>L</sub>, top-quark and ghost, respectively.

discussion. The ring contribution for the Higgs field is given by

$$\begin{aligned}
 V_{\text{ring}}^{\text{Higgs}}(h, T) &\equiv -\frac{1}{2}T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \sum_{l=1}^{\infty} \frac{1}{l} \left( -\frac{1}{\omega_n^2 + \vec{k}^2 + m_h^2(h)} \Pi_h(T) \right)^l \\
 &= -\frac{T}{12\pi} \text{Tr} \left\{ [m_h^2(h) + \Pi_h(T)]^{3/2} - m_h^3(h) \right\}, \tag{A10}
 \end{aligned}$$

where the thermal mass comes from the diagrams in the limit  $m(h)/T \ll 1$  shown in Fig. 5 and becomes

$$\Pi_h(T) = \Pi_{\phi}^{(A_{\mu}^a)}(T) + \Pi_{\phi}^{(B_{\mu})}(T) + \Pi_{\phi}^{(\psi)}(T) + \Pi_{\phi}^{(\phi)}(T) = \frac{T^2}{12} \left( \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 + 3y_t^2 + 6\lambda \right), \tag{A11}$$

with

$$\Pi_{\phi}^{(A_{\mu}^a)}(T) = \frac{3}{16}g_2^2T^2, \quad \Pi_{\phi}^{(B_{\mu})}(T) = \frac{1}{16}g_Y^2T^2, \quad \Pi_{\phi}^{(\psi)}(T) = \frac{1}{4}y_t^2T^2, \quad \Pi_{\phi}^{(\phi)}(T) = \frac{1}{2}\lambda T^2. \tag{A12}$$

Note that these contributions are evaluated by setting the external momentum to zero since we are interested in the infrared limit.

In a similar manner, one can obtain the ring contributions from the gauge bosons, which becomes

$$V_{\text{ring}}^{\text{gb}}(h, T) \equiv -\frac{T}{12\pi} \text{Tr} \left\{ [M^2(h) + \Pi_{00}(T)]^{3/2} - M^3(h) \right\}. \tag{A13}$$

Here the mass matrices in the original gauge field basis  $(A_\mu^i, B_\mu)$  are

$$M^2(h) = \begin{pmatrix} g_2^2 h^2/4 & 0 & 0 & 0 \\ 0 & g_2^2 h^2/4 & 0 & 0 \\ 0 & 0 & g_2^2 h^2/4 & -g_Y g_2 h^2/4 \\ 0 & 0 & -g_Y g_2 h^2/4 & g_2^2 h^2/4 \end{pmatrix}, \quad (\text{A14})$$

$$\Pi_{00}(T) = \begin{pmatrix} \Pi_{00}^{(2)}(T) & 0 & 0 & 0 \\ 0 & \Pi_{00}^{(2)}(T) & 0 & 0 \\ 0 & 0 & \Pi_{00}^{(2)}(T) & 0 \\ 0 & 0 & 0 & \Pi_{00}^{(1)}(T) \end{pmatrix}, \quad (\text{A15})$$

where  $\Pi_{00}(T)$  is the (00) component of the polarization tensor in the infrared limit, namely  $\Pi_{\mu\nu}(p=0, T)$  and

$$\Pi_{00}^{(1)}(T) = \Pi_\phi^{(1)}(T) + \Pi_\psi^{(1)}(T) = \frac{11}{6} g_Y^2 T^2, \quad (\text{A16})$$

$$\Pi_{00}^{(2)}(T) = \Pi_{\text{gb}}^{(2)}(T) + \Pi_\phi^{(2)}(T) + \Pi_\psi^{(2)}(T) = \frac{11}{6} g_2^2 T^2, \quad (\text{A17})$$

with

$$\Pi_\phi^{(1)}(T) = \frac{1}{6} g_Y^2 T^2, \quad \Pi_\psi^{(1)}(T) = \frac{5}{3} g_Y^2 T^2, \quad (\text{A18})$$

$$\Pi_{\text{gb}}^{(2)}(T) = \frac{2}{3} g_2^2 T^2, \quad \Pi_\phi^{(2)}(T) = \frac{1}{6} g_2^2 T^2, \quad \Pi_\psi^{(2)}(T) = g_2^2 T^2. \quad (\text{A19})$$

These thermal masses are obtained by calculating the two-point functions of SU(2) and U(1) gauge fields shown in Fig. 6. Evaluating the eigenvalues of  $M^2(h) + \Pi_{00}(T)$  and  $M^2(h)$  to the three-half power, and then taking trace of them, we have

$$\begin{aligned} V_{\text{ring}}^{\text{gb}}(h, T) = & -\frac{T}{12\pi} \left[ 2a_g^{3/2} + \frac{1}{2\sqrt{2}} (a_g + c_g - [(a_g - c_g)^2 + 4b_g^2]^{1/2})^{3/2} \right. \\ & \left. + \frac{1}{2\sqrt{2}} (a_g + c_g + [(a_g - c_g)^2 + 4b_g^2]^{1/2})^{3/2} - \frac{1}{4} [g_2^2 h^2]^{3/2} - \frac{1}{8} [(g_2^2 + g_Y^2) h^2]^{3/2} \right], \end{aligned} \quad (\text{A20})$$

where  $a_g$ ,  $b_g$  and  $c_g$  are given in Eq. (46).

Note that we have worked in the Landau gauge to evaluate the contributions from the gauge bosons. Although the thermal mass matrix (A15) can be diagonal only in the limit  $m_W(h)/T \ll 1$  and  $m_Z(h)/T \ll 1$ , the ring contribution is still valid for the larger mass,

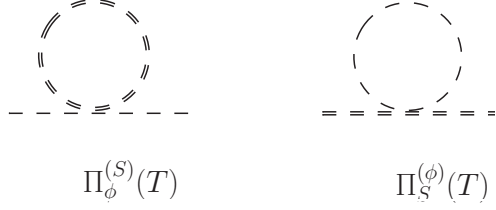


FIG. 7: The diagrams which contribute to the thermal masses of the Higgs field and the new scalar one. The double dashed line denotes the new scalar field  $S$ .

thus the larger field value than temperature. This is because the ring contribution vanishes for the larger mass.

In case where the scalar  $S$  is introduced, the contribution from the diagram shown in Fig. 7 is added, and then the thermal masses of the Higgs field and  $S$  are given as Eq. (44) and (45), respectively.

### 1. The thermal functions and their approximation

The high temperature expansion is often applied to the thermal functions (A8). However, it is not useful for investigating the large field value  $m(\phi)/T \equiv r \geq 1$ . In this subsection we compare the exact forms of the thermal functions (A8) numerically evaluated with their approximated forms and investigate the effectiveness of them.

The thermal functions (A8) with the high temperature expansion become

$$J_B(r^2) \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12}r^2 - \frac{\pi}{6}r^3 - \frac{r^4}{32} \left[ \ln(r^2/16\pi^2) + 2\gamma_E - \frac{3}{2} \right], \quad (\text{A21})$$

$$J_F(r^2) \simeq \frac{7\pi^4}{360} - \frac{\pi^2}{24}r^2 - \frac{r^4}{32} \left[ \ln(r^2/\pi^2) + 2\gamma_E - \frac{3}{2} \right], \quad (\text{A22})$$

where  $\gamma_E \approx 0.57721$  is the Euler's gamma. Besides, it is known that the thermal functions can be fitted by the following functions [22]:

$$J_{B(F)}(r^2) = e^{-r} \sum_{n=0}^{N_{B(F)}} c_n^{B(F)} r^n, \quad (\text{A23})$$

where  $N_{B(F)}$  and  $c_n^{B(F)}$  are the truncation order of the series and the fitting coefficients, respectively. For the fixed truncation order  $N_{B(F)}$ , we find the coefficients  $c_n^{B(F)}$  by fitting the exact results numerically evaluated.

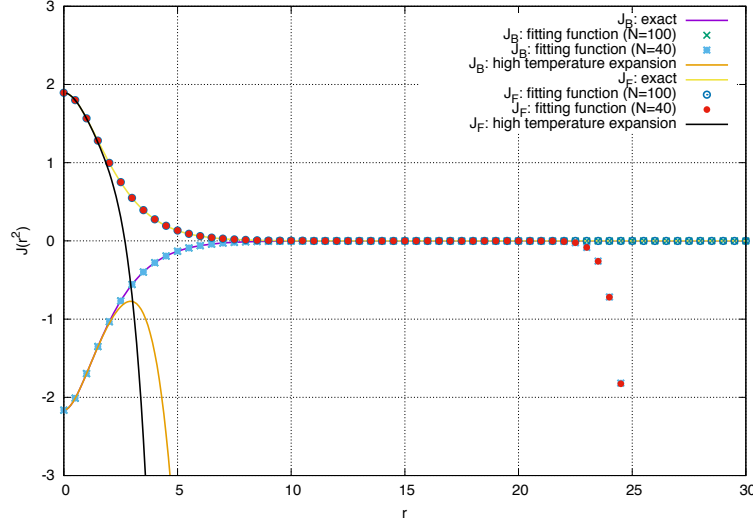


FIG. 8: Comparison of the thermal functions between different approximations.

We show the comparisons between results of exact form Eq. (A8) and the approximated forms Eq. (A21), (A22) and (A23) in Fig. 8. We see that the high temperature expansions are actually valid for  $r \leq 2$  and the fitting functions with  $N_{B(F)} = 40$  break down for  $r > 22$ . The fitting functions with  $N_{B(F)} = 100$  are valid for large value of  $r$ . Therefore, it is useful for evaluating a potential of large field values since  $r = m(\phi)/T$  and the mass  $m(\phi)$  is proportional to the value of the field  $\phi$ .

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